Mathematics of Circular Motion

* Mathematics of Circular Motion$ \frac{}{}$

There are three mathematical quantities that will be of primary interest to us as we analyze the motion of objects in circles. These three quantities are speed, acceleration and force. The speed of an object moving in a circle is given by the following equation.

Ka tre sasi matematike që do të jetë me interes parësor për ne si ne analizojmë lëvizjen e objekteve në qarqet. Këto tri sasitë janë shpejtësia, nxitimi dhe forca.Shpejtësia e një objekti lëviz në një rreth jepet nga ekuacioni i mëposhtëm.

shpejtësia mesatare =$\frac{distanca}{koha}$ = $\frac{2πR}{T}$ , ku R- është rrezja e rrethit, T- është perioda

The acceleration of an object moving in a circle can be determined by either two of the following equations.

Përshpejtimi i një objekti lëviz në një rreth mund të përcaktohet ose nga dy prej ekuacioneve në vijim.

nxitimi = $\frac{v^{2}}{R}$ , nxitimi = $\frac{4∙π^{2}R}{T^{2}} $

The equation on the right (above) is derived from the equation on the left by the substitution of the expression for speed.

The [net force](http://www.physicsclassroom.com/Class/newtlaws/u2l2d.cfm) (**Fnet**) acting upon an object moving in circular motion is directed inwards. While there may by more than one force acting upon the object, the vector sum of all of them should add up to the net force. In general, the inward force is larger than the outward force (if any) such that the outward force cancels and the unbalanced force is in the direction of the center of the circle. The net force is related to the acceleration of the object (as is always the case) and is thus given by the following three equations:

Ekuacioni në të djathtë (më sipër), e ka prejardhjen nga ekuacioni në të majtë me zëvendësimin e shprehjes për shpejtësi.

Forca neto (Fnet) vepron mbi një objekt lëviz në lëvizje rrethore është e drejtuar nga brenda. Përderisa mund të më shumë se një forcë që vepron mbi objektin, shuma vektoriale e të gjitha ato duhet të shtoni deri në forcën neto. Ne pergjithesi, force hyrëse është më e madhe sesa forca e jashtme (nëse ka) i tillë që forca e jashtme anullon dhe force pabalancuar është në drejtim të qendrës së rrethi.Forca neto është e lidhur me përshpejtimin e objektit (siç është gjithmonë rasti) dhe është dhënë në këtë mënyrë nga tre ekuacionet e mëposhtme:



The equations in the middle (above) and on the right (above) are derived from the equation on the left by the substitution of the expressions for acceleration.

This set of circular motion equations can be used in two ways:

* as a ["recipe" for algebraic problem-solving](http://www.physicsclassroom.com/Class/circles/U6L1e.cfm#prob) in order to solve for an unknown quantity.
* as a *guide to thinking* about how an alteration in one quantity would affect a second quantity.

These two ways are illustrated below.

**Equations as a Guide to Thinking**

An equation expresses a mathematical relationship between the quantities present in that equation. For instance, the equation for Newton's second law identifies how acceleration is related to the net force and the mass of an object.



The relationship expressed by the equation is that the acceleration of an object is directly proportional to the net force acting upon it. In other words, the bigger the net force value is, the bigger that the acceleration value will be. As net force increases, the acceleration increases. In fact, if the net force were increased by a factor of 2, the equation would predict that the acceleration would increase by a factor of 2. Similarly, if the net force were decreased by a factor of 2, the equation would predict that the acceleration would decrease by a factor of 2.

Newton's second law equation also reveals the relationship between acceleration and mass. According to the equation, the acceleration of an object is inversely proportional to mass of the object. In other words, the bigger the mass value is, the smaller that the acceleration value will be. As mass increases, the acceleration decreases. In fact, if the mass were increased by a factor of 2, the equation would predict that the acceleration would decrease by a factor of 2. Similarly, if the mass were decreased by a factor of 2, the equation would predict that the acceleration would increase by a factor of 2.

As mentioned previously, equations allow for predictions to be made about the affect of an alteration of one quantity on a second quantity. Since the Newton's second law equation shows three quantities, each raised to the first power, the predictive ability of the equation is rather straightforward. The predictive ability of an equation becomes more complicated when one of the quantities included in the equation is raised to a power. For instance, consider the following equation relating the net force (**Fnet**) to the speed (**v**) of an object moving in uniform circular motion.



This equation shows that the net force required for an object to move in a circle is directly proportional to the square of the speed of the object. For a constant mass and radius, the **Fnet** is proportional to the **speed2**.



The factor by which the net force is altered is the square of the factor by which the speed is altered. Subsequently, if the speed of the object is doubled, the net force required for that object's circular motion is quadrupled. And if the speed of the object is halved (decreased by a factor of 2), the net force required is decreased by a factor of 4.

**Equations as a Recipe for Problem-Solving**

The mathematical equations [presented above](http://www.physicsclassroom.com/Class/circles/U6L1e.cfm#p1) for the motion of objects in circles can be used to solve circular motion problems in which an unknown quantity must be determined. The process of solving a circular motion problem is much like any other problem in physics class. The process involves a careful reading of the problem, the identification of the known and required information in variable form, the selection of the relevant equation(s), substitution of known values into the equation, and finally algebraic manipulation of the equation to determine the answer. Consider the application of this process to the following two circular motion problems.

|  |
| --- |
| **Sample Problem #1**A 900-kg car moving at 10 m/s takes a turn around a circle with a radius of 25.0 m. Determine the acceleration and the net force acting upon the car. |

The solution of this problem begins with the identification of the known and requested information.

|  |  |
| --- | --- |
| **Known Information:**m = 900 kgv = 10.0 m/sR = 25.0 m | **Requested Information:**a = ????Fnet = ???? |

To determine the acceleration of the car, use the equation a = v2/ R. The solution is as follows:

a = v2 / R

a = (10.0 m/s)2 / (25.0 m)

a = (100 m2/s2) / (25.0 m)

**a = 4 m/s2**

To determine the net force acting upon the car, use the equation Fnet = m•a. The solution is as follows.

Fnet = m • a

Fnet = (900 kg) • (4 m/s2)

**Fnet = 3600 N**

|  |
| --- |
| **Sample Problem #2**A 95-kg halfback makes a turn on the football field. The halfback sweeps out a path that is a portion of a circle with a radius of 12-meters. The halfback makes a quarter of a turn around the circle in 2.1 seconds. Determine the speed, acceleration and net force acting upon the halfback. |

The solution of this problem begins with the identification of the known and requested information.

|  |  |
| --- | --- |
| **Known Information:**m = 95.0 kgR = 12.0 mTraveled 1/4-th of the circumference in 2.1 s | **Requested Information:**v = ????a = ????Fnet = ???? |

To determine the speed of the halfback, use the equation v = d / t where the d is one-fourth of the circumference and the time is 2.1 s. The solution is as follows:

v = d / t

v = (0.25 • 2 • pi • R) / t

v = (0.25 • 2 • 3.14 • 12.0 m) / (2.1 s)

**v = 8.97 m/s**

To determine the acceleration of the halfback, use the equation a = v2 / R. The solution is as follows:

a = v2/ R

a = (8.97 m/s)2 / (12.0 m)

a = (80.5 m2/s2) / (12.0 m)

**a = 6.71 m/s2**

To determine the net force acting upon the halfback, use the equation Fnet = m•a. The solution is as follows.

Fnet = m\*a

Fnet = (95.0 kg)\*(6.71 m/s2)

**Fnet = 637 N**

In Lesson 2 of this unit, circular motion principles and the above mathematical equations will be combined to explain and analyze a variety of real-world motion scenarios including [amusement park rides](http://www.physicsclassroom.com/Class/circles/u6l2b.cfm) and [circular-type motions in athletics](http://www.physicsclassroom.com/Class/circles/u6l2c.cfm).

**Check Your Understanding**

1. Anna Litical is practicing a centripetal force demonstration at home. She fills a bucket with water, ties it to a strong rope, and spins it in a circle. Anna spins the bucket when it is half-full of water and when it is quarter-full of water. In which case is more force required to spin the bucket in a circle? Explain using an equation as a "guide to thinking."

**See Answer**

2. A Lincoln Continental and a Yugo are making a turn. The Lincoln is four times more massive than the Yugo. If they make the turn at the same speed, then how do the centripetal forces acting upon the two cars compare. Explain.

**See Answer**

3. The Cajun Cliffhanger at Great America is a ride in which occupants line the perimeter of a cylinder and spin in a circle at a high rate of turning. When the cylinder begins spinning very rapidly, the floor is removed from under the riders' feet. What affect does a doubling in speed have upon the centripetal force? Explain.

**See Answer**

4. Determine the centripetal force acting upon a 40-kg child who makes 10 revolutions around the Cliffhanger in 29.3 seconds. The radius of the barrel is 2.90 meters.

**See Answer**

Motion rrethore dhe gravitacionit

Lëvizja rrethore

Ky grup i 27 problemeve synon aftësinë tuaj për të kombinuar ligjet e Njutonit dhe lëvizje dhe gravitacionit ekuacionet rrethore në mënyrë që të analizojë lëvizjen e objekteve që lëvizin në qarqet, duke përfshirë satelitët rrotullohen. Problemet varg në vështirësi nga shumë e lehtë dhe drejt-përpara për të shumë e vështirë dhe komplekse. Problemet më të vështira janë ngjyra-koduar si probleme blu.

Karakteristikat lëvizje e objektet lëvizin në qarqet

Objektet që lëvizin në qarqet kanë një shpejtësi e cila është e barabartë me distancën udhëtuar për kohën e udhëtimit.Distance rreth një rreth është e barabartë me një qark dhe llogaritur si 2 • pi • R ku R eshte rrezja.Koha për një revolucion rreth rrethit është referuar si periudhë dhe shënohet me simbolin T. Kështu shpejtësia mesatare e një objekti në lëvizje rrethore është dhënë nga shprehja e 2 • pi • R / T. Shpesh herë deklarata problemi ofronfrekuenca rrotulluese në revolucione për minutë apo revolucione për sekondë. Çdo revolucion rreth rrethit është e barabartë me një perimetër të distancës. Kështu, shumëzuar frekuencën rrotulluese me perimetrin lejon një për të përcaktuar shpejtësinë mesatare e objektit.

Përshpejtimi i objekteve që lëvizin në qarqet bazohet kryesisht mbi një ndryshim drejtim.Shkalla aktuale Përshpejtimi varet se sa shpejt drejtim është duke u ndryshuar dhe është e lidhur direkt me shpejtësi dhe anasjelltas lidhur me rreze të kthesës. Ai përfundon se nxitimi është dhënë nga shprehja v2 / R, ku v është shpejtësia dhe R është rrezja e rrethit.

Ekuacionet për shpejtësi mesatare (v) dhe përshpejtimin mesatare (a) janë përmbledhur më poshtë.

v = d / t = 2 • pi • R / T = frekuenca • 2 • pi • R

a = V2 / R